

Treatise on Caustics, Torsion and Shear in Spacetime:

Introduction

I've always believed that the best theories emerge from a deep dive into history—not just to avoid reinventing the wheel, but to spot the gaps where new ideas can take root. This treatise is my attempt to trace the mathematical evolution of concepts like torque (a torque-like stiffness that keeps propagation finite), shear (the cutting resistance allowing coherent intersections), torsion (the twist in spacetime connections), and caustics (those focusing singularities where paths converge). These aren't just abstract notions; they're the building blocks that prevent physics from falling into instantaneous, acausal messes, much like how my Void-Motion Spacetime (VMS) framework uses an effective space-tension T_s as an analogy to keep the geometry causal—consistent with a finite c —without introducing any new dimensional scale beyond $S_0 = \hbar$.

Drawing from primary sources spanning Euclid to modern gravitational-wave analyses, I'll weave a narrative that highlights how non-zero structural invariants have been indispensable for causal dynamics. In VMS, torque and shear manifest as fundamental asymmetries: T_s for the torque-like stiffness of space and a shear term tied to S in harmonic closures. Caustics, controlled by these invariants, dictate how voids focus and disperse, linking to loop stability in my model.

Definitions (for consistency with my calibration papers). I use T_s for torque-like stiffness and τ exclusively for torsion. 'Tear' is the small but non-zero closure defect ϵ . The six calibration levers $\kappa, \tau, \chi, \epsilon, \varphi, \beta$ are translators of these two asymmetries (torque and tear) into the measurements we actually have. Shear in this essay is the same notion captured by my functional $S[\{\gamma_i\}]$ in the math appendix. No new dimensional scales are introduced beyond $S_0 = \hbar$; everything else is dimensionless or purely geometric.

1. Ancient Foundations: Euclidean Geometry and Newtonian Mechanics

Our story starts with the basics of space as flat and absolute. Euclid's *Elements* (c. 300 BCE) laid the groundwork for geometry, assuming an infinite, uniform plane where measurements are straightforward—no twists, no shears, just straight lines and angles. This view held for centuries until Isaac Newton formalized mechanics in *Philosophiæ Naturalis Principia Mathematica* (1687). Newton introduced torque as $\tau = r \times F$, the cross-product capturing rotational force, and used calculus to describe motion. Yet, his gravity was instantaneous action-at-a-distance, a mathematical convenience that clashed with emerging ideas of finite speeds—no built-in resistance to prevent infinite propagation.

This era's limitation? Space lacked intrinsic “stiffness”—no torque-like factor to enforce causality, no shear to handle transverse disturbances. It's a reminder of how VMS introduces T_s early on (as an effective stiffness analogy) to avoid such pitfalls, keeping the framework consistent with finite-speed propagation without positing a material medium.

Key citation: Newton, I. (1687). *Philosophiæ Naturalis Principia Mathematica*. London: Royal Society.

2. The Ether Era and Finite Propagation (1800s–Early 1900s)

The 19th century brought the ether as a medium for light, introducing shear and torque analogies. Augustin-Jean Fresnel's *Mémoire sur la diffraction de la lumière* (1818) modeled light as transverse waves in an elastic ether, implying it must support shear resistance (like a solid resisting deformation) to transmit oscillations without longitudinal modes. James Clerk Maxwell's *A Treatise on Electricity and Magnetism* (1873) unified EM, deriving finite-speed waves via the stress tensor, embedding causality in field theory.

Maxwell stress tensor (vacuum, SI units): $T_{ij} = \epsilon_0 (E_i E_j - \frac{1}{2} \delta_{ij} E^2) + (1/\mu_0)(B_i B_j - \frac{1}{2} \delta_{ij} B^2)$

Hendrik Lorentz (1895–1904) reconciled this with mechanics through transformations. Historical note: “ether” was a scaffold; modern EM does not require a medium. I use elasticity analogies strictly as analogies.

This era foreshadowed VMS: space needs non-zero invariants like an effective T_s for finite-speed consistency and shear for coherent caustic intersections—purely as structural analogies, not an ontic medium.

References:

- A. Fresnel, *Mémoire sur la diffraction de la lumière* (1818).
- J.C. Maxwell, *A Treatise on Electricity and Magnetism* (1873). Oxford: Clarendon Press.
- H.A. Lorentz, *Electromagnetic phenomena in a system moving with any velocity less than that of light* (1904).

3. Non-Euclidean Geometry, Tensor Calculus, and Riemannian Curvature

Carl Friedrich Gauss's *Disquisitiones generales circa superficies curvas* (1827) introduced intrinsic curvature, independent of embedding—space could bend without external reference. Bernhard Riemann's 1854 habilitation *Über die Hypothesen, welche der Geometrie zu Grunde liegen* generalized to n-dimensions with the curvature tensor:

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu} \Gamma^{\rho}_{\nu\sigma} - \partial_{\nu} \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda} \Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda} \Gamma^{\lambda}_{\mu\sigma}$$

Gregorio Ricci-Curbastro and Tullio Levi-Civita's tensor calculus (1880s–1890s; *Méthodes de calcul différentiel absolu*, 1900) enabled curved-space descriptions. This toolkit was crucial for GR and for VMS, where curvature $\kappa(s)$ in small deformations ties to torsion/shear in loop stability.

Key citations:

- Gauss, C.F. (1827). *Disquisitiones generales circa superficies curvas*. Göttingen.
- Riemann, B. (1854). *Über die Hypothesen, welche der Geometrie zu Grunde liegen*. Habilitationsvortrag, Göttingen.

- Ricci-Curbastro, G., & Levi-Civita, T. (1900). Méthodes de calcul différentiel absolu et leurs applications. Mathematische Annalen.

4. Einstein's Relativity and the Geometric Revolution (1905–1915)

Albert Einstein's Zur Elektrodynamik bewegter Körper (1905) eliminated ether, making c a symmetry principle in special relativity. In Die Feldgleichungen der Gravitation (1915), general relativity equated gravity to curvature:

$$R_{\{\mu\nu\}} - \frac{1}{2} R g_{\{\mu\nu\}} + \Lambda g_{\{\mu\nu\}} = (8\pi G/c^4) T_{\{\mu\nu\}}$$

Finite-speed propagation arises from hyperbolic field equations; shear and curvature are built into geometry. This shift made space dynamic. (VMS heuristic: voids may induce curvature deficits $\Delta g(r) \sim 1/r^2$ —VMS-specific and not GR standard.)

References:

- A. Einstein, Zur Elektrodynamik bewegter Körper (1905).
- A. Einstein, Die Feldgleichungen der Gravitation (1915).

5. Mid-20th Century: Shear, Rotation, and Torsion Extensions (1950s–1970s)

Amal Kumar Raychaudhuri's 1955 equation for congruences separates expansion θ , shear σ^2 , and rotation ω^2 . In a standard timelike form:

$$\dot{\theta} = - (1/3) \theta^2 - \sigma_{\{ab\}} \sigma^{\{ab\}} + \omega_{\{ab\}} \omega^{\{ab\}} - R_{\{ab\}} u^a u^b \quad (\text{signs/factors per convention})$$

The shear term drives focusing/caustics. Stephen Hawking and Roger Penrose used this structure for singularity theorems (Proc. R. Soc. A 314, 529–548, 1970), showing shear leads to inevitable focusing under certain conditions.

Élie Cartan introduced the torsion tensor $T^{\lambda}_{\{\mu\nu\}} = \Gamma^{\lambda}_{\{\mu\nu\}} - \Gamma^{\lambda}_{\{\nu\mu\}}$. Einstein–Cartan theory (Sciama 1962, Kibble 1961) couples torsion to spin. Extensions add torsion to Raychaudhuri, inducing relative accelerations in congruences (e.g., Luz & Vitagliano 2017). In VMS, this resonates: torsion in small deformations stabilizes loops; shear in caustics ensures coherent void resolutions.

References:

- A.K. Raychaudhuri, Relativistic Cosmology. I. Phys. Rev. 98, 1123 (1955).
- S.W. Hawking & R. Penrose, The Singularities of Gravitational Collapse and Cosmology, Proc. R. Soc. A 314, 529–548 (1970).
- É. Cartan (1922). Sur une généralisation de la notion de courbure de Riemann et les espaces à torsion. C. R. Acad. Sci. Paris.
- T.W.B. Kibble, Lorentz invariance and the gravitational field, J. Math. Phys. 2, 212 (1961).
- D.W. Sciama (1962). On the analogy between charge and spin in general relativity, in Recent Developments in General Relativity.
- P. Luz & V. Vitagliano, Raychaudhuri Equation in Spacetimes with Torsion, Phys. Rev. D 96, 024021 (2017).

6. Late 20th–21st Century: Elasticity Analogies and Caustics

B. Carter & H. Quintana (1972) formalized relativistic elasticity; R. Beig & B.G. Schmidt (2002) cast it hyperbolic for finite speeds. T. Padmanabhan (2004) framed Einstein equations as elasticity. Battye & Pearson (2013) linked massive gravity to elasticity (rigidity view). Erik Verlinde's emergent gravity (2016) ties to elastic entropy perspectives. K.T. McDonald (2018, Princeton) discussed orders-of-magnitude effective stiffness of spacetime inferred from gravitational-wave strains (qualitative inference; estimates vary).

Caustics: A.O. Petters, H. Levine, and J. Wambsganss' Singularity Theory and Gravitational Lensing (2001) defines caustics via a mapping Jacobian with $\det(\partial y/\partial x) = 0$ —loci of formal infinite magnification. In Raychaudhuri language, shear drives the approach to such focusing sets.

For VMS, the torque-like stiffness T_s (historical elasticity analogy) keeps propagation consistent with finite speed; shear (as captured by S) enables harmonic closures; caustics (regulated by the balance of shear/torque) allow coherent void intersections instead of singular collapse.

References:

- B. Carter & H. Quintana, Proc. R. Soc. A 331, 57 (1972).
- R. Beig & B.G. Schmidt, Relativistic Elasticity, arXiv:gr-qc/0211054 (2002).
- T. Padmanabhan, Gravity as Elasticity of Spacetime, arXiv:gr-qc/0408051 (2004).
- R.A. Battye & J.A. Pearson, Phys. Rev. D 88, 084004 (2013), arXiv:1301.5042.
- E. Verlinde, Emergent Gravity and the Dark Universe, arXiv:1611.02269 (2016).
- K.T. McDonald, What is the Stiffness of Spacetime? Princeton (2018).
- A.O. Petters, H. Levine, J. Wambsganss, Singularity Theory and Gravitational Lensing, Birkhäuser (2001).

Reflection and Call to Contribute

Looking back, this history shows non-zero invariants aren't optional—they're the guardrails for causality. From Newton's torque to Cartan's torsion, Riemann's curvature to Raychaudhuri's shear, and Fresnel's elasticity to modern GW stiffness, each advance reinforced that space needs resistance-like structure to function physically. In VMS, I've echoed this: T_s as torque-like stiffness for finite void motion, shear for coherent caustics in loops. It works, but it's not finished—mathematicians, if this resonates, let's talk. Can we formalize S in Raychaudhuri terms for VMS caustics? How does torsion extend void stability beyond small deformations? Review, critique, contribute—email or comment on Zenodo. Together, we can refine this into something useful.

(Version 1.0, September 20, 2025—open for iterations based on feedback.)

Full References

- Euclid. (c. 300 BCE). Elements.
- Newton, I. (1687). Philosophiæ Naturalis Principia Mathematica. London: Royal Society.

- A. Fresnel, *Mémoire sur la diffraction de la lumière* (1818).
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